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Magnetic susceptibility of the QCD vacuum at finite quark-chemical potential

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Abstract

We investigate the QCD magnetic susceptibility χ at the finite quark-chemical potential ($\mu \neq 0$) and at zero temperature ($T = 0$) to explore the pattern of the magnetic phase transition of the QCD vacuum. For this purpose, we employ the nonlocal chiral quark model derived from the instanton vacuum in the presence of the chemical potential in the chiral limit. Focusing on the Nambu-Goldstone phase, we find that the magnetic susceptibility remains almost stable to $\mu \approx 200$ MeV, and falls down drastically until the the quark-chemical potential reaches the critical point $\mu_c \approx 320$ MeV. Then, the strength of the χ is reduced to be about a half of that at $\mu = 0$, and the first-order magnetic phase transition takes place, corresponding to the chiral restoration. From these observations, we conclude that the response of the QCD vacuum becomes weak and unstable to the external electromagnetic field near the critical point, in comparison to that for vacuum. It is also shown that the breakdown of Lorentz invariance for the magnetic susceptibility, caused by the finite chemical potential, turns out to be small.

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I. INTRODUCTION

It is of great importance to understand nontrivial structures of the QCD vacuum, since it reflects strong nonperturbative fluctuations and lay the foundation for explaining low-energy phenomena of hadrons. In particular, the quark condensate characterises the non-perturbative features of the QCD vacuum and plays a role of an order parameter associated with spontaneous breakdown of chiral symmetry (SBχS) which is known to be essential in low-energy hadronic phenomena. The nature of the QCD vacuum can be examined under external fields such as electromagnetic (EM) field, since it reveals the EM properties of the vacuum. The magnetic susceptibility is defined as the vacuum expectation value (VEVs) in Euclidean space:

$$\langle iq^\dagger \sigma_{\mu\nu} q\rangle_F = ie_q F_{\mu\nu}\langle iq^\dagger q\rangle \chi, \tag{1}$$

where e_q denotes the electric charge of the quark and $F_{\mu\nu}$ represents the EM field-strength tensor. The $\langle iq^\dagger q\rangle$ stands for the quark (chiral) condensate taken as a normalization. It is natural to take it as a normalization for the magnetic susceptibility, since the VEV $\langle iq^\dagger \sigma_{\mu\nu} q\rangle_F$ breaks chiral symmetry [1]. Recently, Braun et al. [2] have proposed that the χ may be measured in the exclusive photoproduction of hard dijets $\gamma + N \to q\bar{q} + N$. Theoretically, there have been already many works on it in the QCD sum rules and chiral effective models [3, 4, 5, 6, 7, 8, 9]. The magnetic susceptibility is also related to the photon distribution amplitude [5].

While the magnetic susceptibility is relatively well understood in free space, it is not much known about its modification at finite density (finite quark-chemical potential, μ). For example, there are several interesting theoretical points as follows: First, in principle, the pattern of the *magnetic* phase transition of QCD vacuum can be revealed by studying the change of the magnetic susceptibility in medium [10]. Second, one can see how the vacuum reacts in the presence of the external EM source, especially, near the critical point. Third, the medium modification of the χ may play a critical role in explaining the exclusive photoproduction of the hard dijets in nuclei. Finally, this information may be important in understanding the EM features of neutron stars [11]. Moreover, since Lorentz invariance is broken in medium, we must consider separately the space and time components of the magnetic susceptibility at finite quark-chemical potential. In fact, Refs. [12, 13, 14] have studied the importance of the breakdown of Lorentz invariance for the in-medium pion weak

decay constant F_π. In fact, Ref. [15] has calculated the magnetic susceptibility without the breakdown of Lorentz invariance taken into account. In addition, the chemical potential was treated perturbatively in Ref. [15]. We will consider explicitly the breakdown of Lorentz invariance in the present work.

In order to compute the χ at finite quark-chemical potential, we employ the framework of the μ-modified nonlocal chiral quark model (NLχQM) derived from the instanton vacuum [16]. The NLχQM has several virtues: All relevant QCD symmetries are satisfied within the model. In particular, the instanton vacuum well realizes spontaneous breakdown of chiral symmetry via quark zero modes. Moreover, there are only two parameters: the average size of instantons ($\bar{\rho} \approx 1/3$ fm) and average inter-instanton distance ($\bar{R} \approx 1$ fm), which can be determined by the internal constraint such as the self-consistent equation [17, 18, 19]. These values for ρ and R have been supported in various LQCD simulations recently [20, 21, 22]. There is no further adjustable parameter in the model. Note that the inverse of the average size of instantons, $\bar{\rho}^{-1} \simeq 600$ MeV, is regarded as the normalization point of the present model, so that we can calculate the dependence of the magnetic susceptibility on the chemical potential to the critial point ($\mu \approx 320$ MeV) without any trouble.

We concentrate in this work on the Nambu-Goldstone (NG) phase. In order to go beyond the NG phase, we need subtle techniques which we will explain briefly later. As a result, we find that the χ remains stable up to $\mu \approx 200$ MeV, and falls down drastically to the critical quark-chemical potential, $\mu_c \approx 320$ MeV and the strength of the χ is reduced to be about a half of that at $\mu = 0$. The *first-order* magnetic phase transition takes place at this critical point corresponding to the phase of the chiral restoration. We will show that the response of the QCD vacuum becomes weak and unstable to the external EM source near the critical point. Moreover, the breakdown of Lorentz invariance turns out to be small in contrast to that in the case of the pion weak decay constant F_π [12].

We organize the present work as follows: In Section II, we briefly explain the μ-modified NLχQM. In Section III, we compute the magnetic susceptibility χ and present the numerical results with discussions. The final Section summarises the present work and draws conclusions. The outlook of this work is also shortly discussed in this Section.

II. NONLOCAL CHIRAL QUARK MODEL AT FINITE QUARK-CHEMICAL POTENTIAL

A modified Dirac equation with the finite quark-chemical potential μ in (anti)instanton ensemble can be written in Euclidean space as follows:

$$[i\partial\!\!\!/ - i\hat{\mu}\!\!\!/ + A\!\!\!/_{I\bar{I}}]\,\Psi^{(n)}_{I\bar{I}} = \lambda_n \Psi^{(n)}_{I\bar{I}}, \tag{2}$$

where $\hat{\mu}_\alpha = (0,0,0,\mu)$. Note that we consider here the chiral limit $(m \to 0)$. The subscript $(\bar{I})I$ stands for the (anti)instanton contribution, and we use a singular-gauge instanton solution:

$$A^\alpha_\mu(x) = \frac{2\bar{\eta}^{\alpha\nu}_\mu \bar{\rho}^2 x_\nu}{x^2(x^2+\bar{\rho}^2)}, \tag{3}$$

where $\eta^{\alpha\nu}_\mu$ and $\bar{\rho}$ denote the 'tHooft symbol and average instanton size ($\bar{\rho} \approx 1/3$ fm), respectively. Since the instanton has its own spatial distribution with finite size (ρ) in principle, one needs to consider an instanton-distribution function when the integral over the collective coordinate for the instanton (instanton center, color orientation, and instanton size) is performed [17, 18, 19]. However, we assume here a simple δ-fucntion type for the insatanton-distribution function, i.e., $\delta(\rho-\bar{\rho})$. In fact, the effect of the finite size is taken as $1/N_c$ corrections [18, 19]. We refer to Ref. [9] for the finite-size effect on the magnetic susceptibility in free space. The quark zero-mode solution then can be obtained by solving the following equation:

$$[i\partial\!\!\!/ - i\hat{\mu}\!\!\!/ + A\!\!\!/_{I\bar{I}}]\,\Psi^{(0)}_{I\bar{I}} = 0. \tag{4}$$

The explicit form of $\Psi^{(0)}$ can be found in Refs. [16, 23]. Assuming that the low-energy hadronic properties are dominated by the zero mode, we can write the quark propagator in one instanton background approximately as follows:

$$S_{I\bar{I}}(x,y) \;=\; \langle\psi(x)\psi(y)^\dagger\rangle = -\sum_n \frac{\Psi^{(n)}_{I\bar{I}}(x)\Psi^{(n)\dagger}_{I\bar{I}}(y)}{\lambda_n + im} \approx S_0(x,y) - \frac{\Psi^{(0)}_{I\bar{I}}(x)\Psi^{(0)\dagger}_{I\bar{I}}(y)}{im}, \tag{5}$$

where S_0 is a free quark propagator, $S_0 = (i\partial\!\!\!/ - i\hat{\mu}\!\!\!/)^{-1}$. Incorporating Eqs. (3) and (5), and averaging over all instanton collective coordinates, we can obtain the following expression for the quark propagator in momentum space:

$$S(p,\mu) = -\frac{1}{p\!\!\!/ + i\hat{\mu}\!\!\!/ - iM(p,\hat{\mu})}. \tag{6}$$

The momentum- and μ-dependent effective quark mass $M(p,\hat{\mu})$, corresponding to the Fourier transform of the quark zero-mode solution of Eq. (4), reads:

$$M(p,\hat{\mu}) = M_0(\hat{\mu})\,[(p+i\hat{\mu})^2\psi^2(p,\hat{\mu})], \tag{7}$$

where M_0 designates the constituent-quark mass as a function of $\hat{\mu}$, which will be determined self-consistently within the model. The analytical expression for $\psi(p,\hat{\mu})$ is also given in Ref. [16]. In actual calculations, we make use of a dipole-type parameterization, instead of using Eq. (7), for numerical simplicity:

$$M(p,\hat{\mu}) \;\approx\; M_0(\hat{\mu})\left[\frac{2\Lambda^2}{(p+i\hat{\mu})^2+2\Lambda^2}\right]^2. \tag{8}$$

The cutoff mass Λ corresponds to the inverse of the average (anti)instanton size ($\sim 1/\bar{\rho}$), resulting in $\Lambda \approx 600$ MeV. We have verified that this simple parameterization reproduces Eq. (7) qualitatively well [24]. One can refer to Ref. [25] for more quantitative studies on the parameterization of $M(p,\hat{\mu})$.

Now we are in a position to consider the effective partition function for $N_f = 2$ from the instanton vacuum configuration with the finite μ in the large N_c limit. Choosing the relevant terms for further discussion, it can be written as follows [16]:

$$\mathcal{Z}_{\rm eff} \;=\; \int d\lambda\, D\psi\, D\psi^\dagger \exp\left[\int d^4x\, \psi^\dagger(i\partial\!\!\!/ - i\mu\!\!\!/ + im)\psi + \lambda(Y^+ + Y^-) - N\ln\lambda\right], \tag{9}$$

where the flavour, colour, and Dirac spin indices are assumed tacitly. Note that this effective partition function has been constructed in such way that we can obtain the quark propagator given in Eq. (5). $Y^\pm$ denotes the $2N_f$-'tHooft interaction in the (anti)instanton background at finite μ. As for the parameters appearing in the second and third terms of Eq. (9), λ plays a role of the Lagrangian multiplier, whereas N indicates the sum of the number of the (anti)instantons. For more details on this partition function and its derivation, one may refer to Ref. [16], and references therein.

We would like to explain briefly the phase structure obtained via the present model. Since both the quark-antiquark and quark-quark interactions for $N_f = 2$ are attractive, two possible phases arise: The Nambu-Goldstone (NG) and colour-superconducting (CSC) phases. They are characterised by the QCD order paramters, i.e., the chiral and diquark condensates, respectively, or by the finite constituent quark mass M_0 and diquark energy gap Δ, equivalently. As shown in Ref. [16], solving the Dyson-Schwinger-Gorkov (DSG)

equation and using the instanton packing fraction $N/V \approx (200\,\mathrm{MeV})^4$, one can compute M_0 and Δ as functions of μ by minimizing the partition function with respect to λ. In the present work, we consider only the pure NG phase, in which the magnetic susceptibility is well defined in terms of the chiral condensate as shown in Eq. (1). We ignore, for simplicity, the metastable mixed phases. As a result, the value of the critical chemical potential (μ_c) was determined to be about 320 MeV. We note that it is about 5% different from that given in Ref. [16] in which the renormalization scale is taken to be slightly different from ours, $\Lambda \approx 1/\bar{\rho} \approx 0.6$ GeV.

III. QCD MAGNETIC SUSCEPTIBILITY AT FINITE μ

In this Section, we compute the VEV of the tensor current of Eq. (1) in the the background EM field in the presence of the finite μ and provide numerical results for the magnetic susceptibility χ. In order to deal with the background EM field, we employ the Schwinger method [6, 29]. Then, we can write the covariant effective partition function with an external tensor source T as follows:

$$\mathcal{Z}_{\mathrm{eff}}[T,\mu] \;=\; \int d\lambda\, D\psi\, D\psi^\dagger \exp\left[\sum_{f=1}^{2}\int d^4x\,\psi_f^\dagger(i\not{D} - i\not{\mu} + \sigma\cdot T)\psi_f + \lambda(Y_2^+ + Y_2^-)\right]. \quad (10)$$

where we have written only the relevant terms to compute the VEV. iD_μ indicates the covariant derivative, $i\partial_\mu + e_q V_\mu$, in which e_q and V_μ are the quark electric charge and external photon field, respectively. The $\sigma_{\mu\nu}$ is the well-known spin antisymmetric tensor defined as $\sigma_{\mu\nu} = i(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)/2$, and $\sigma\cdot T$ denotes $\sigma_{\mu\nu}T^{\mu\nu}$. Differentiating Eq. (10) with respect to the external source T, we can evaluate the VEV as follows:

$$\langle i\psi^\dagger\sigma_{\mu\nu}\psi\rangle_F = \frac{1}{VN_f}\frac{\partial \ln \mathcal{Z}_{\mathrm{eff}}[T,\hat{\mu}]}{\partial T_{\mu\nu}}\bigg|_{T=0} = i\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}_{c,\gamma}[S(P,\hat{\mu})\,\sigma_{\mu\nu}], \quad (11)$$

where P stands for $p - e_qV$. Equation (11) can be evaluated further straightforwardly by expanding with respect to the electric charge to order $\mathcal{O}(e_q)$ as follows:

$$S(P,\mu)\,\sigma_{\mu\nu} = \frac{1}{\bar{\not{P}} + iM(\bar{P})}\sigma_{\mu\nu} = -\frac{\frac{ie_q}{2}\sigma\cdot F + i[\not{P}, M(\bar{P})]}{[\bar{p}^2 + M^2(\bar{p})]^2}[\bar{\not{p}} - iM(\bar{p})]\sigma_{\mu\nu} + \mathcal{O}(e_q^2). \quad (12)$$

In evaluating Eq. (12), we have used the simplified notation $\bar{p} = p + i\hat{\mu}$ and the identity $[P_\mu, P_\nu] = ie_qF_{\mu\nu}$. The commutation relation in Eq. (12) reads:

$$[\not{P}, M(\bar{P})] \;=\; \gamma_\nu[P_\nu, M(\bar{P})] = -2ie_q\tilde{M}'(\bar{p})\,(p_\mu F_{\mu\nu} + i\mu F_{4\nu})\gamma_\nu. \quad (13)$$

Note that there appears a pure electric part proportional to $F_{4\nu}$ in the above equation, due to the breakdown of Lorentz invariance at finite μ in addition to the usual EM field-strength tensor, $F_{\mu\nu}$. The $\tilde{M}'$ designates the derivative of the effective-quark mass with respect to the momentum squared:

$$\tilde{M}'(\bar{p}) = \frac{\partial M(\bar{p})}{\partial p^2}. \tag{14}$$

For convenience, we call the terms proportional to $\tilde{M}'$ as a nonlocal contribution, since they arise from the nonlocal (derivative) interaction, whereas the terms without it a local one. Considering all the ingredients discussed so far, and performing the trace of Eq. (11) over the colour and Dirac spin spaces, we obtain as follows:

$$\begin{aligned}
\mathrm{Tr}_{c,\gamma}[S\sigma_{\mu\nu}] &= \frac{4N_c e_q M(\bar{p})}{[\bar{p}^2 + M^2(\bar{p})]^2} F_{\mu\nu} - \frac{8N_c e_q \tilde{M}'(\bar{p})}{[\bar{p}^2 + M^2(\bar{p})]^2} \left[\bar{p}_\mu \bar{p}_\sigma F_{\sigma\nu} - (\mu \leftrightarrow \nu)\right] \\
&\approx \underbrace{\frac{4N_c e_q M(\bar{p})}{[\bar{p}^2 + M^2(\bar{p})]^2} F_{\mu\nu} - \frac{8N_c e_q \tilde{M}'(\bar{p})}{[\bar{p}^2 + M^2(\bar{p})]^2} \left[p_\mu p_\sigma F_{\sigma\nu} - p_\nu p_\sigma F_{\sigma\mu}\right]}_{\text{magnetic+electric}} \\
&- \underbrace{\frac{8N_c e_q \tilde{M}'(\bar{p})}{[\bar{p}^2 + M^2(\bar{p})]^2} \left[\mu^2(\delta_{\nu 4} F_{4\mu} - \delta_{\mu 4} F_{4\nu})\right]}_{\text{electric}}.
\end{aligned} \tag{15}$$

Here, we have ignored the terms proportional to $\mathcal{O}(p)$ in evaluation of Eq. (15), since they become negligible according to the integral identity $\int d^4p\, p_\mu f(p^2) = 0$.

First, we turn off the pure electric part in Eq. (15) for simplicity, resulting in the magnetic contribution to the magnetic susceptibility χ, assigned as χ_{M}. Incorporating Eqs. (11) and (15), and then equating with Eq. (1), we finally arrive at the following compact relation for χ_{M}:

$$\chi_{\mathrm{M}} \langle i q^\dagger q \rangle = 4N_c \int \frac{d^4p}{(2\pi)^4} \left[\frac{M(\bar{p}) - \bar{p}^2 \tilde{M}'(\bar{p})}{[\bar{p}^2 + M^2(\bar{p})]^2}\right]. \tag{16}$$

One can easily see that Eq. (16) is a generalized expression for Eq. (52) given in Ref. [6] by replacing $p \to p + i\mu$. Switching on the pure electric part, we can write the electric contribution to the magnetic susceptibility as χ_{E} with an additional nonlocal term, coming from the breakdown of Lorentz invariance, as follows:

$$\chi_{\mathrm{E}} \langle i\psi^\dagger \psi \rangle = \chi_{\mathrm{M}} \langle i\psi^\dagger \psi \rangle + \underbrace{4N_c \int \frac{d^4p}{(2\pi)^4} \left[\frac{\mu^2 \tilde{M}'(\bar{p})}{[\bar{p}^2 + M^2(\bar{p})]^2}\right]}_{\text{Breakdown of Lorentz invariance}}. \tag{17}$$

In Fig. 1, we present the numerical results for the $\chi_{\mathrm{M,E}} \langle i\psi^\dagger \psi \rangle$ (left panel) and the $\chi_{\mathrm{M,E}}$ (right panel) as functions of μ, respectively. The magnetic contribution is drawn in the solid

curve, whereas the electric one in the dashed one. As for the $\chi_{\rm M,E}\langle i\psi^\dagger\psi\rangle$ depicted in the left panel, we show each contribution (local, nonlocal and total) separately. Note that the dominant local contributions are the same for the magnetic and electric contributions as expected from Eqs. (16) and (17), and starts to decrease beyond $\mu\approx 200$ MeV. At $\mu=\mu_c$, its strength is reduced to about a half of that at $\mu=0$. On the contrary, the nonlocal contributions are almost flat for the NG phase, and show *small* difference between the magnetic and electric ones, caused by the breakdown of Lorentz invariance, being different from the pion weak decay constant [12, 13, 14]. As a result, the total contributions resemble the local one. Moreover, the *first-order* magnetic phase transition occurs at the critical chemical potential, following the chiral restoration, as shown in Fig. 1. Several model calculations, using the nonlocal chiral quark model [6] and QCD sum rules [3, 4, 5], computed $\chi\langle i\psi^\dagger\psi\rangle$ at $\mu=0$. We note that our value ~ 46 MeV is in good agreement with those in Refs. [3, 4, 5].

Finally, in the right panel of Fig. 1, we show the numerical results for the $\chi_{\rm M,E}$ in the same manner as in the left panel. Since the chiral condensate is rather flat for the NG phase with respect to μ as shown in Refs. [12, 16], the overall tendency of the $\chi_{\rm M,E}$ is very similar to that of the $\chi_{\rm M,E}\langle i\psi^\dagger\psi\rangle$. Again, we observe small difference between the $\chi_{\rm M}$ and $\chi_{\rm E}$, and the first-order magnetic phase transition at the critical point. Note that in the vicinity of μ_c the $\chi_{\rm M,E}$ drecreases by about a factor of two in magnitude compared to its value at $\mu=0$. As mentioned previously, since the Fermi surface begins to be filled just beyond $\mu\approx 300$ MeV, which corresponds approximately to the normal nuclear density $\rho_0\approx 0.17\,{\rm fm}^{-3}$, we expect that there is a rapid change of the χ near ρ_0. In other words, the response of the QCD vacuum becomes weak but unstable to the external EM source as the density is closed to ρ_0, while it remains relatively stable otherwise.

In Ref. [11], the colour-symmetric one-gluon exchange (OGE) was employed to investigate the χ in medium in addition to the static screening effect, and was found that a sharp divergence takes place in the χ near the normal nuclear density, although it depends on the screening and the current-quark mass. This observation may be consistent with the present results showing the first-order magnetic phase transition.

In fact, partial results of the magnetic susceptibility at finite quark-chemical potential has been already studies within the same framework [15]. However, there are several important points that were not considered in Ref. [15]. Firstly, the breaking of Lorentz invariance was assumed to be small, so that it was ignored. Secondly, the chemical potential was

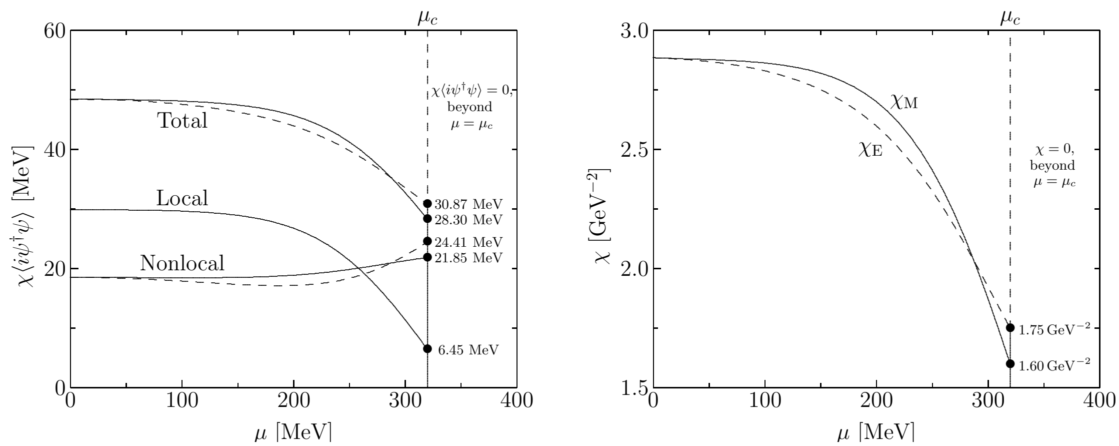

FIG. 1: $\chi_{\rm M,E}\langle i\psi^\dagger\psi\rangle$ (left panel) and $\chi_{\rm M,E}$ (right panel) as functions of μ. The magnetic contribution is presented in the solid curve, whereas the electric one in the dashed one. The vertical dashed lines indicate the $\mu_c \approx 320$ MeV.

treated perturbatively, so that the calculation was performed by expanding it for numerical simplicity. Unfortunately, this naive expansion has led to the wrong behavior of the magnetic susceptibility, in particular, as μ is getting close to its critical value. Finally, the finite value of the current quark mass was taken into account in Ref. [15]. However, we soon realized that a mere extension of the model by Carter and Diakonov [16] seems to be unjustified. Because of this reason, we are restricted ourselves to consider the chiral limit in the present work.

IV. SUMMARY AND CONCLUSIONS

In the present work, we have investigated the QCD magnetic susceptibility χ induced by the external constant electromagnetic field at the finite quark-chemical potential, i.e. $\mu \neq 0$ at $T = 0$. We focussed on the Nambu-Goldstone (NG) phase, in which the χ is well defined. We employed the μ-modified nonlocal chiral quark model derived from the instanton vacuum at the normalization scale $\Lambda = \bar{\rho}^{-1} \approx 600$ MeV, corresponding to the phenomenological instanton parameters $\bar{R} \approx 1$ fm and $\bar{\rho} \approx 1/3$ fm.

We started with the μ-modified Dirac equation to obtain the quark propagator in the instanton ensemble. The μ-dependent effective quark mass can be obtained from the Fourier transform of the quark-zero mode solution. However, we employed the parameterized dipole-type form factor instead of it to circumvent numerical difficulties. The effective partition

function was constructed in such way to reproduce the quark propagator in the presence of the instanton background. Solving the Dyson-Schwinger-Gorkov equation, as done in Ref. [16], we were able to explore the phase structure through two order parameters, M_0 and Δ, corresponding to the Nambu-Goldstone and colour-superconducting phases, respectively. As a result, the critical quark-chemical potential μ_c was determined to be about 320 MeV.

Using the low-energy effective partition function with the finite quark-chemical potential, we computed the magnetic susceptibility of the QCD vacuum χ. We found that the χ remains stable up to $\mu \approx 200$ MeV, and drops then drastically. At the critical quark-chemical potential, $\mu_c \approx 320$ MeV, the strength of the χ is decreased by about a factor of two, compared to that at $\mu = 0$, and the first-order magnetic phase transition takes place, corresponding to the chiral restoration. From these observations, we conclude that the response of the QCD vacuum becomes weak (insensitive) and unstable to the external electromagnetic source near the normal nuclear density in comparison to that for the vacuum. Moreover, the effect of the breakdown of Lorentz invariance, caused by the finite μ, on the χ turns out to be small, $\chi_{\rm M} \sim \chi_{\rm E}$: We note that this tendency is rather different from the pion weak decay constant. Related works including explicit flavour SU(3) symmetry breaking effects as well as meson-loop corrections at finite quark-chemical potential are under progress.

We want at this point to mention that we have calculated the magnetic susceptibility in the NG phase only, that is, below the critical quark-chemical potential. Note that there is, however, one caveat: In principle, we can compute the VEV in Eq. (1) beyond the critical quark-chemical potential, i.e., in the CSC phase. We have to keep in mind that the SU(3) colour symmetry is broken down to SU(2) $\times$ U(1) in the CSC pahse, which causes a lift of colour degeneracy. Thus, the quark propagator must be separated into two different quark propagators of which each consists of 4×4 propagators in the chiral L-R basis [16]. The form of the propagators have been obtained by solving the Schwinger-Dyson-Gorkov equation with a systematic expansion in parameters $1/N_c$ and $\bar{\rho}/\bar{R}$. Taking consideration of these propagators, we can compute the magnetic susceptibility beyond the NG phase, though the calculation is quite involved.

Last but not least, the magnetic susceptibility defined in Eq.(1) is not the magnetization in medium as done in Ref. [11]. The magnetization can be rewritten as the VEV of the following operator $\langle iq^\dagger\gamma_4\sigma_{\mu\nu}q\rangle_F$. This is another interesting quantity which we can compute

within the present scheme. The related investigations are under way.

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